Chiral extrapolations of lattice QCD results

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Outline

Chiral extrapolations: an opportunity

- Extrapolations (the continuum, the chiral and the infinite volume) are rather an opportunity than a nuisance
- Especially the chiral extrapolation allows us to learn something about QCD which we can not get from phenomenology
- In doing this we also check the lattice calculations against some firm predictions of CHPT (the chiral logs)

Chiral extrapolations: an opportunity

- Extrapolations (the continuum, the chiral and the infinite volume) are rather an opportunity than a nuisance
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- In doing this we also check the lattice calculations against some firm predictions of CHPT (the chiral logs)

But

- the chiral region may be rather small (we start entering it now)
- the necessary precision and control over systematic effects may be very high

Pion mass and decay constant

$$M_\pi^2 = M^2 \left\{ 1 - \frac{1}{2} x \ln \frac{\Lambda_3^2}{M^2} + \frac{17}{8} x^2 \left(\ln \frac{\Lambda_M^2}{M^2} \right)^2 + x^2 k_M + O(x^3) \right\}$$

$$F_\pi = F \left\{ 1 + x \ln \frac{\Lambda_4^2}{M^2} - \frac{5}{4} x^2 \left(\ln \frac{\Lambda_F^2}{M^2} \right)^2 + x^2 k_F + O(x^3) \right\}$$

where $\quad x = \frac{M^2}{(4\pi F)^2} \qquad M^2 = 2Bm$

$\Lambda_{3,4}$ are low energy constants (LEC) in form of energy scales:

$$\bar{\ell}_n = \ln \frac{\Lambda_n^2}{M_\pi^2} \qquad n = 1, \ldots, 7$$

Pion mass and decay constant

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where $x = \frac{M^2}{(4\pi F)^2}$ $\quad M^2 = 2Bm$

$\Lambda_{M,F}$ are combinations thereof

$$\ln \frac{\Lambda_M^2}{M^2} = \frac{1}{51} \left(28 \ln \frac{\Lambda_1^2}{M^2} + 32 \ln \frac{\Lambda_2^2}{M^2} - 9 \ln \frac{\Lambda_3^2}{M^2} + 49 \right)$$

$$\ln \frac{\Lambda_F^2}{M^2} = \frac{1}{30} \left(14 \ln \frac{\Lambda_1^2}{M^2} + 16 \ln \frac{\Lambda_2^2}{M^2} + 6 \ln \frac{\Lambda_3^2}{M^2} - 6 \ln \frac{\Lambda_4^2}{M^2} + 23 \right)$$

Pion mass and decay constant

$$M_\pi^2 = M^2 \left\{ 1 - \frac{1}{2} x \ln \frac{\Lambda_3^2}{M^2} + \frac{17}{8} x^2 \left(\ln \frac{\Lambda_M^2}{M^2} \right)^2 + x^2 k_M + O(x^3) \right\}$$

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$$\text{where} \qquad x = \frac{M^2}{(4\pi F)^2} \qquad M^2 = 2Bm$$

and $k_{M,F}$ $O(p^6)$ LEC

Pion mass and decay constant – Numerics

Numbers from the phenomenology: $F = 86.2 \pm 0.5$ MeV

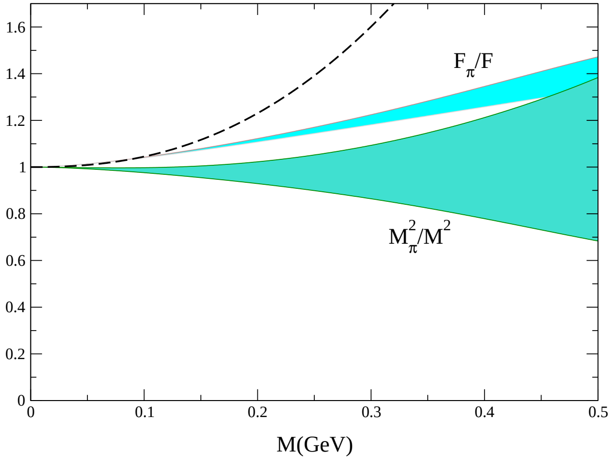
$$\begin{aligned}
\bar{\ell}_1 &= -0.4 \pm 0.6 &\Leftrightarrow\quad \Lambda_1 &= 0.12\,^{+0.04}_{-0.03}\ \text{GeV} \\
\bar{\ell}_2 &= 4.3 \pm 0.1 &\Leftrightarrow\quad \Lambda_2 &= 1.2 \pm 0.06\ \text{GeV} \\
\bar{\ell}_3 &= 2.9 \pm 2.4 &\Leftrightarrow\quad \Lambda_3 &= 0.6\,^{+1.4}_{-0.4}\ \text{GeV} \\
\bar{\ell}_4 &= 4.4 \pm 0.2 &\Leftrightarrow\quad \Lambda_4 &= 1.25\,^{+0.15}_{-0.13}\ \text{GeV}
\end{aligned}$$

From these, it follows:

$$\Lambda_M = 0.6\,^{+0.18}_{-0.15}\ \text{GeV} \qquad \Lambda_F = 0.5\,^{+0.16}_{-0.13}\ \text{GeV}$$

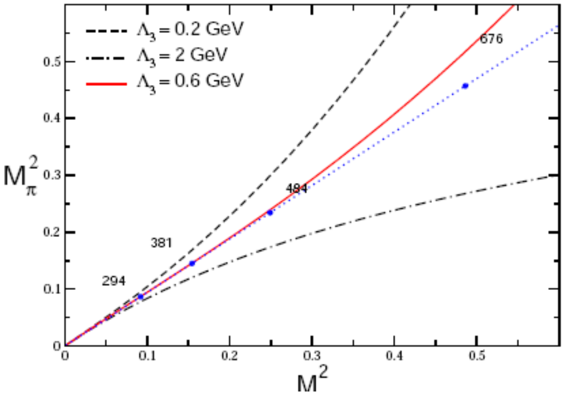
The NNLO LEC's k_F and k_M can only be roughly estimated

Pion mass and decay constant – Plots



From CG, Gasser, Leutwyler (01)

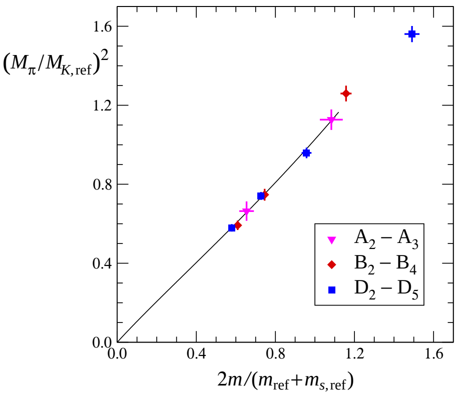
Pion mass and decay constant – Plots



Lattice data from Del Debbio et al.

Thanks to H. Leutwyler for the figure

Recent lattice results



Del Debbio et al. (06)

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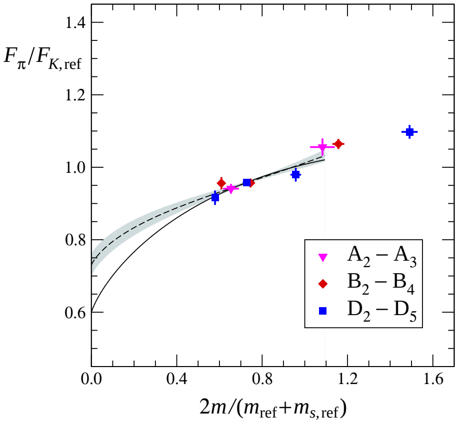

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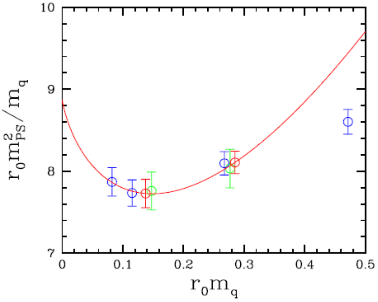
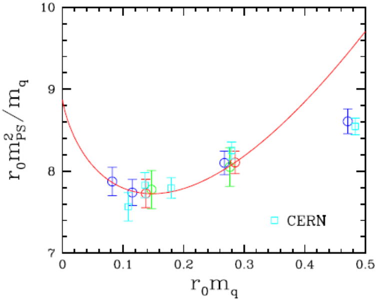
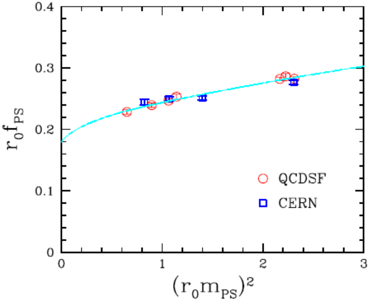
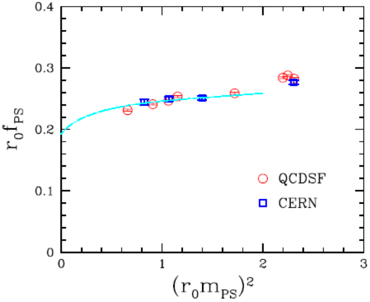




Schierholz, talk at Lat07

Recent lattice results



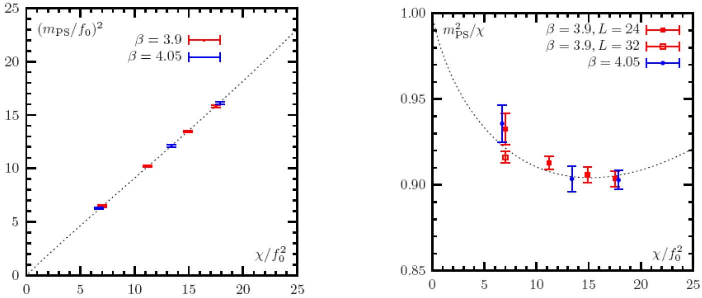


$r_0 f_0 = 0.179(2) \quad r_0\Lambda_4 = 3.32(6)$

$r_0 f_0 = 0.192(3) \quad r_0\Lambda_4 = 3.32(6)$

Schierholz, talk at Lat07

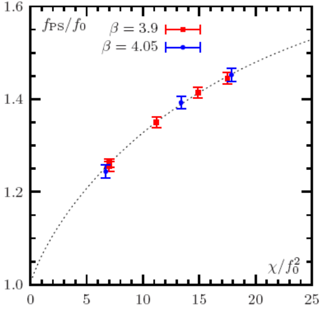
Recent lattice results



Urbach, talk at Lat07

Recent lattice results

Pion-Sector: f_{PS} as function of the Quark Mass



- fit: $\chi^2/\mathrm{dof} = 1.2$
- lattice spacings:

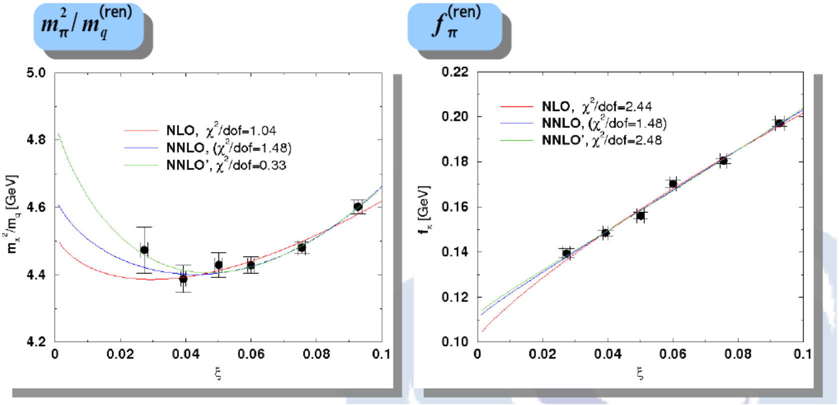
$$a(3.90) = 0.0855(5)(3)\ \mathrm{fm}$$
$$a(4.05) = 0.0666(6)(9)\ \mathrm{fm}$$

using $f_\pi = 130.7$ MeV and $m_\pi^0 = 135$ MeV [Aubin et al., 2004]

$\rightarrow$ $r_0 = 0.441(14)(5)$ fm

Urbach, talk at Lat07

Recent lattice results



Noaki, talk at Lat07

Summary of recent lattice results

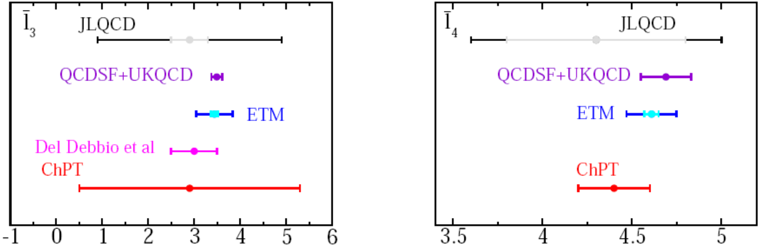
Summary of the lattice results from recent $N_f = 2$ simulations ('06,'07):

	F (MeV)	$\Sigma^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})$ $(\mathrm{MeV})^3$	$\bar{l}_3$	$\bar{l}_4$
Del Debbio et al			3.0(5)	
ETM	85.98(7)(21)(35)	266(6)(0)(6)	3.44(8)(26)(6)	4.61(4)(3)(7)
QCDSF/UKQCD	79(5)	273(12)	3.49(12)	4.69(14)
JLQCD	78.1(2.7)(1.2)	242(6)(6)	2.9(4)(1.6)	4.3(5)(2)

Talk by S. Necco at Lattice 07

Summary of recent lattice results

Results for $\bar{l}_3, \bar{l}_4$ from $N_f = 2$ simulations:



Talk by S. Necco at Lattice 07

Chiral predictions for a_0^0 and a_0^2

$$
\begin{aligned}
a_0^0 &= 0.220 \pm 0.001 + 0.027\Delta_{r^2} - 0.0017\Delta\ell_3 \\
10 \cdot a_0^2 &= -0.444 \pm 0.003 - 0.04\Delta_{r^2} - 0.004\Delta\ell_3
\end{aligned}
$$

where

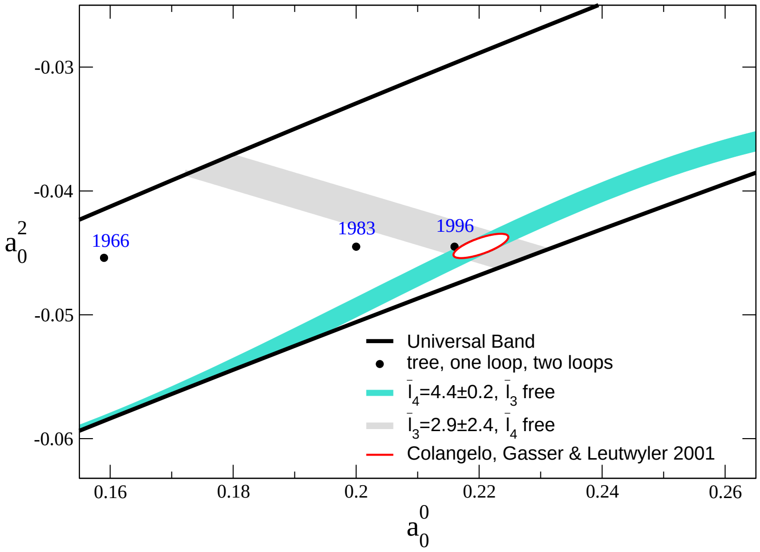
$$
\langle r^2\rangle_s = 0.61\mathrm{fm}^2(1+\Delta_{r^2}) \qquad \bar{\ell}_3 = 2.9 + \Delta\ell_3
$$

with $\langle r^2\rangle_s = \frac{3}{8\pi^2 F_\pi^2}\left\{\bar{\ell}_4 - \frac{13}{12} + \xi\Delta_r + \mathcal{O}(\xi^2)\right\}$ $\qquad \xi \equiv \left(\frac{M_\pi^2}{4\pi F_\pi^2}\right)$

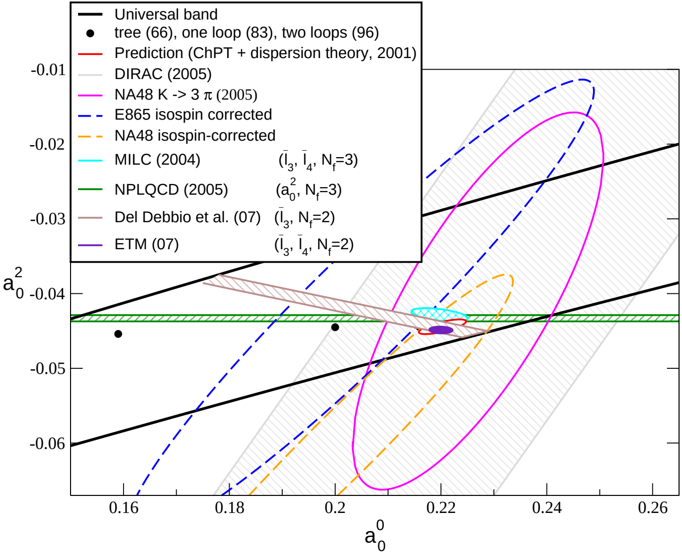
Adding errors in quadrature $\qquad [\Delta_{r^2} = 6.5\%,\ \Delta\ell_3 = 2.4]$

$$
\begin{aligned}
a_0^0 &= 0.220 \pm 0.005 \\
10 \cdot a_0^2 &= -0.444 \pm 0.01 \\
a_0^0 - a_0^2 &= 0.265 \pm 0.004
\end{aligned}
$$

Chiral predictions for a_0^0 and a_0^2



The $\pi\pi$ S-wave scattering lengths plane



Sensitivity to the quark condensate

The constant $\bar{\ell}_3$ determines the NLO quark mass dependence of the pion mass

$$M_\pi^2 = 2B\hat{m}\left[1 + \frac{2B\hat{m}}{16\pi F_\pi^2}\bar{\ell}_3 + \mathcal{O}(\hat{m}^2)\right]$$

$$\hat{m} = \frac{m_u + m_d}{2} \qquad B = -\frac{1}{F^2}\langle 0|\bar{q}q|0\rangle$$

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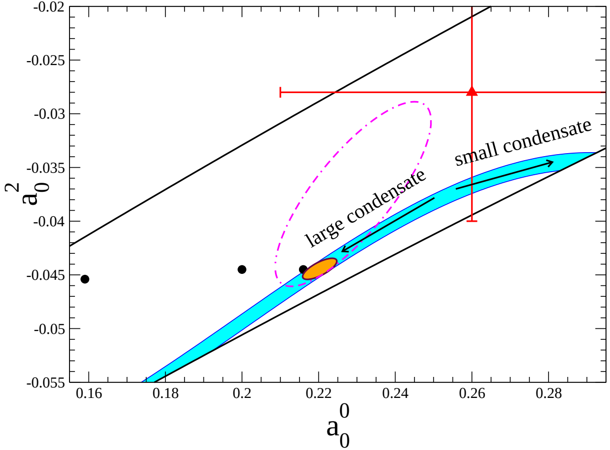
Its size tells us what fraction of the pion mass is given by the Gell-Mann–Oakes–Renner term

$$M^2_{\mathrm{GMOR}} \equiv 2B\hat{m}$$

or how large is the quark condensate, the order parameter of chiral symmetry breaking – as emphasized since long by Jan Stern and collaborators

Sensitivity to the quark condensate



The E865 data on $K_{\ell 4}$ imply that

GC, Gasser and Leutwyler PRL (01)

$$M_{\text{GMOR}} > 94\% M_\pi$$

Recently confirmed by NA48 data on K_{e4} and $K \to 3\pi$

Masses and decay constants in $SU(3)$

$$M_K^2 = {M_K^0}^2 \Bigg\{ 1 + \frac{2}{3}\mu_\eta + \frac{8{M_K^0}^2}{F_0^2}\left[(2L_8^r - L_5^r) + (2 + x_{\pi K})(2L_6^r - L_4^r)\right] + \mathcal{O}\left(M^4\right) \Bigg\}$$

$$F_K = F_0 \left\{ 1 - \frac{3}{4}\mu_\pi - \frac{3}{2}\mu_K - \frac{3}{4}\mu_\eta + \frac{4{M_K^0}^2}{F_0^2}\left[L_5^r + (2 + x_{\pi K})L_4^r\right] + \mathcal{O}\left(M^4\right) \right\}$$

where

$$x_{\pi K} = \left(\frac{M_\pi^0}{M_K^0}\right)^2 \qquad \mu_P = \frac{M_P^2}{32\pi^2 F_P^2}\ln\frac{M_P^2}{\mu^2}$$

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$$F_K = F_0 \left\{ 1 - \frac{3}{4}\mu_\pi - \frac{3}{2}\mu_K - \frac{3}{4}\mu_\eta + \frac{4{M_K^0}^2}{F_0^2}\left[L_5^r + (2 + x_{\pi K})L_4^r\right] + \mathcal{O}\left(M^4\right) \right\}$$

Two-loop formulae for the $SU(3)$ masses and decay constants are also available

Bijnens et al.

Masses and decay constants in $SU(3)$ – Numerics

New preliminary results from MILC, RBC-UKQCD and PACS-CS: L_i at the scale M_ρ

LEC $\cdot 10^3$	MILC	RBC-UKQCD	PACS-CS
$(2L_8 - L_5)$	0.3(1)(1)	0.247(45)	-0.23(5)
$(2L_6 - L_4)$	$0.3(1)\left(^{+2}_{-3}\right)$	-0.02(42)	0.10(4)
L_4	$0.1(3)\left(^{+3}_{-1}\right)$	0.136(80)	-0.02(11)
L_5	$1.4(2)\left(^{+2}_{-1}\right)$	0.862(99)	1.47(15)

Talk by S. Necco at Lattice 07

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cf. fits of Bijnens et al. (2000) $[L_4^r \equiv L_6^r \equiv 0]$

$\mathcal{O}(p^4)$: $2L_8^r - L_5^r = 0.5 \cdot 10^{-3}$ $L_8^r = 1.0 \cdot 10^{-3}$

$\mathcal{O}(p^6)$: $L_5^r = (0.97 \pm 0.11) \cdot 10^{-3}$ $L_8^r = (0.60 \pm 0.18) \cdot 10^{-3}$

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Talk by S. Necco at Lattice 07

The exact values of L_4^r and L_6^r are important to establish what is the role of the strange quark in chiral symmetry breaking, and the m_s-dependence of quantities like F and the chiral condensate – surprises in the m_s expansion? cf. talk by S. Descotes-Genon at Lat07

Nucleon mass

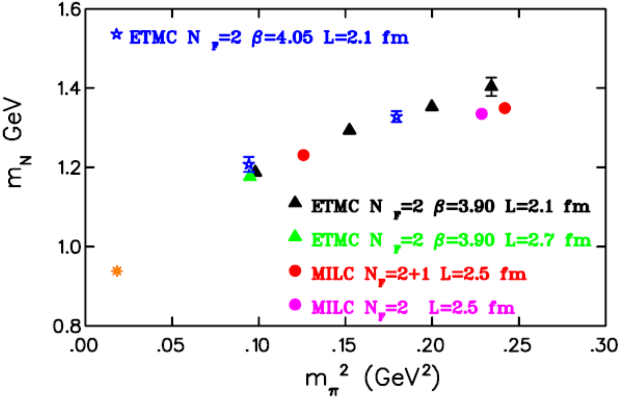
Quark mass dependence known up to $\mathcal{O}(M^4)$:

$$m_N = m - 4c_1 M^2 - \frac{3g^2 M^3}{32\pi F^2} + \left[\tilde{e}_1 - \frac{3(2g^2 - c_2 m)}{8NF^2}\right] M^4 + \mathcal{O}(M^5)$$

The chiral log hidden in $\tilde{e}_1$ is known, but the LEC e_1^r not

Nucleon mass

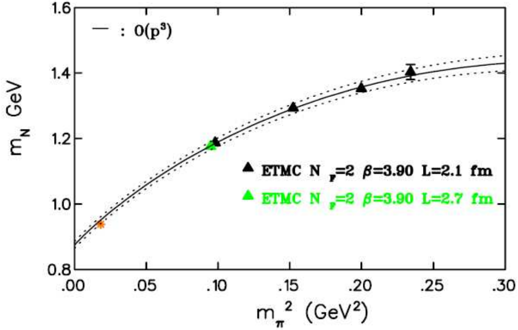
Take $a_{\beta=3.9} = 0.0855$ fm and $a_{\beta=4.05} = 0.0666$ fm



Alexandrou, talk at LAT07

Nucleon mass

Take $a = 0.0855$ fm



- Third order:

$$m_N = m_0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{32\pi f_\pi^2} m_\pi^3$$

- Provides a very good fit with $m_0 = 0.875(10)$ GeV and $c_1 = -1.231(17)$ GeV^{-1} to be compared with $c_1 = -0.9$ GeV^{-1} extracted from $\pi - N$-sigma term.
- Physical point, which is not included in the fit, is reproduced.

Alexandrou, talk at LAT07

Chiral expansion of pion form factors

$$\langle \pi^i(p_1)|\bar{q}\Gamma_L q|\pi^j(p_2)\rangle = C_L F_L(s) \qquad s = (p_1+p_2)^2$$

$$\begin{aligned} L = S \quad &\Rightarrow \quad \Gamma_S = 1\,, \quad C_S = \delta^{ij} \\ L = V \quad &\Rightarrow \quad \Gamma_V = \tau_3\gamma_\mu\,, \quad C_V = i\epsilon^{i3j}(p_1+p_2)_\mu \end{aligned}$$

Taylor expansion near $s = 0$:

$$F_L(s) = F_L(0)\left[1 + \frac{\langle r^2\rangle_L}{6}s + \mathcal{O}(s^2)\right]$$

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Taylor expansion near $s = 0$:

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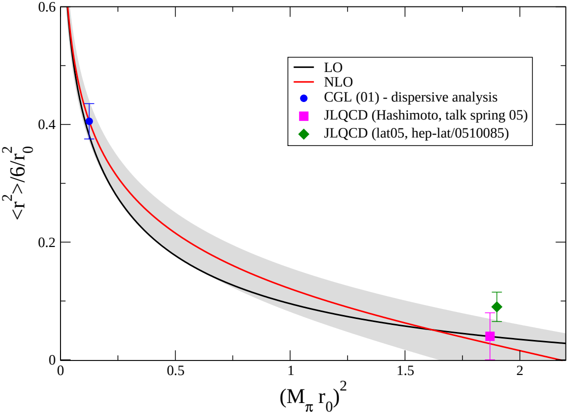
Chiral expansion of the scalar radius:

$$\langle r^2\rangle_S = 6\,\xi\left\{\ln\frac{\Lambda_4^2}{M_\pi^2} - \frac{13}{12} - \frac{29}{18}\,\xi\left(\ln\frac{\Omega_S^2}{M_\pi^2}\right)^2 + \xi\,c_S + \mathcal{O}(\xi^2)\right\}$$

LO: Gasser, Leutwyler (84) NLO: GC, Finkemeier, Urech(96), Bijnens, GC, Talavera (98)

Chiral extrapolation of $\langle r^2 \rangle_{S,V}$



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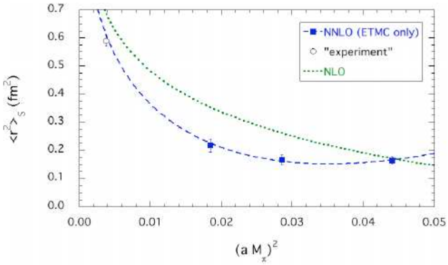


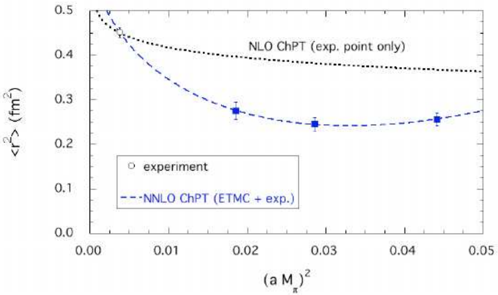
$$\langle r^2 \rangle_S (NLO) = \frac{12}{(4\pi F)^2}\left[\log\left(\frac{\Lambda_4^2}{M_\pi^2}\right) - \frac{13}{12}\right]$$

fix $aF = 0.0534$ and $\overline{\ell}_4 = 4.52$ from PLB '07

$$\langle r^2 \rangle_S (NNLO) = \langle r^2 \rangle_S (NLO) + d_1 M_\pi^2 + d_2 M_\pi^2 \log\left(M_\pi^2\right)$$

Simula, talk at LAT07

Chiral extrapolation of $\langle r^2 \rangle_{S,V}$



$$\langle r^2 \rangle (NLO) = \frac{2}{(4\pi F)^2}\left[\log\left(\frac{\Lambda_6^2}{M_\pi^2}\right) - 1\right]$$

fix $aF = 0.0534$ from PLB '07

exp. $\Rightarrow \bar{\ell}_6 = 14.4\ (3)$

$$\langle r^2 \rangle (NNLO) = \langle r^2 \rangle (NLO) + c_1 M_\pi^2 + c_2 M_\pi^2 \log\left(M_\pi^2\right)$$

ETMC + exp. $\Rightarrow \bar{\ell}_6 = 17.2\ (7)$

Simula, talk at LAT07

πK vector form factor

In order to extract V_{us} from $K_{\ell 3}$ decays, information on the πK vector form factor at $t = 0$ is needed:

$$f_+^{\pi K}(0) = 1 + f_2 + f_4 + \mathcal{O}(p^6)$$

Ademollo-Gatto theorem $\Rightarrow$ f_2 a known function of meson masses:

$$f_2 = -0.023$$

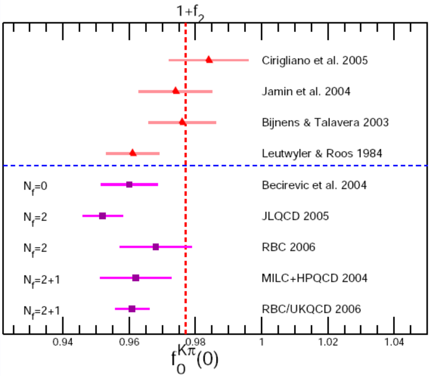
Gasser and Leutwyler (85)

$f_+^{\pi K}(0)$ also known to two loops

Post-Schilcher (01), Bijnens-Talavera (03)

Problem: determine the relevant LEC's

πK vector form factor

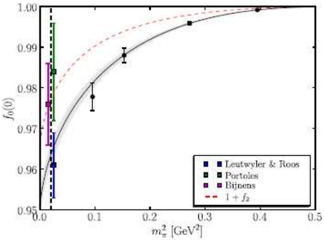


- tension between χPT and lattice?
  $\rightarrow$ need to reduce error bars
- seems independent of N_f
- in many cases Leutwyler & Roos (1984!) is still used to determine $|V_{us}|$ (cf. PDG)

Jüttner, talk at Lat07

πK vector form factor

- Better $\chi^{\rm PT}$ guidance would help



- Published results cover only 3 of four data points

$$f_+^{K\pi}(0) = 0.9609(51)$$

$$|V_{us}| = 0.2257(9)_{\rm exp}(12)_{f_+(0)}$$

- Lightest data point will enhance control of chiral limit
  - Data set complete ; final analysis & systematic estimates in progress
- Results very much favour Leutwyler-Roos with reduced error
- Big impact on V_{us}

Boyle, talk at Lat07

Accurate chiral extrapolation is essential – two-loop formulae complicated $\Rightarrow$ close collaboration between lattice and chiral people

Nucleons: g_A

Chiral expansion known up to $\mathcal{O}(M^3)$:

$$g_A = g + \left(4\tilde{d}_{16} - \frac{g^3}{NF^2}\right) M^2 + \frac{3(1+g^2) - 4m(c_3 - 2c_4)}{24\pi m F^2} g M^3 + \mathcal{O}(M^4)$$

Leading chiral logs at $\mathcal{O}(M^4)$ have also been evaluated

Bernard, Meißner (06)

However the chiral expansion behaves badly already for the physical value of the pion mass: Kambor, Mojziš (99)

$$\frac{-4m(c_3 - 2c_4)}{24\pi m F^2} M^3_{|_{M=0.14\mathrm{GeV}}} \sim 12\%$$

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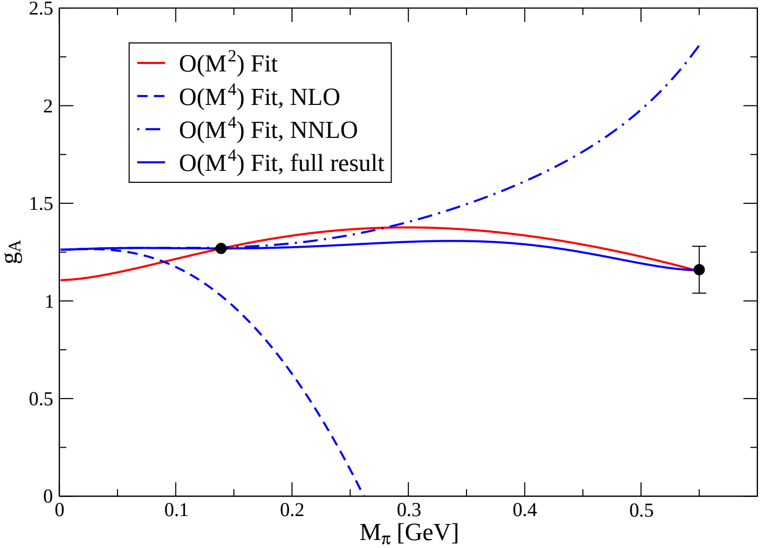
Including the Δ as an explicit degree of freedom tames the growth of the M^3 term Hemmert, Procura, Weise (03)

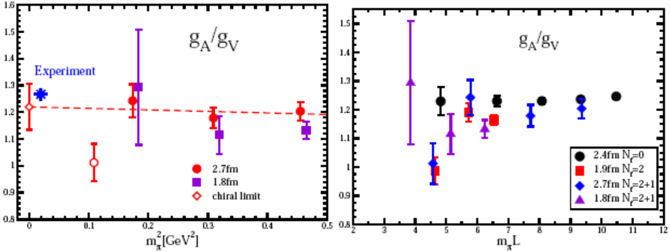
But at the price of a fine tuning

Nucleons: g_A


O(M2) Fit
O(M4) Fit, NLO
O(M4) Fit, NNLO
O(M4) Fit, full result
gA
Mπ [GeV]
0
0.1
0.2
0.3
0.4
0.5
0.5
1
1.5
2
2.5

Nucleons: g_A

- finite volume effect on g_A at lightest mass?
- Chiral limit: $g_A/g_V = 1.220(85)$ omitting lightest mass



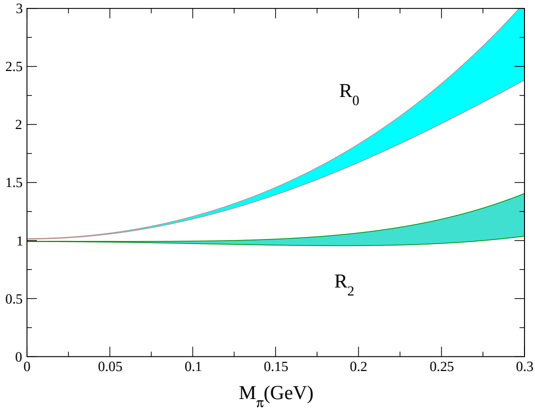
Boyle, talk at Lat07

Chiral extrapolation of a_0^0 and a_0^2

Formulae known to two loops — Bijnens, GC, Ecker, Gasser and Sainio (95)

Warning: chiral convergence of the $\pi\pi$ scattering lengths is bad

The relevant mass scale is rather $2M_\pi$!



$R_I = a_0^I / a_0^I(\mathrm{LO})$ — GC, Gasser and Leutwyler (01)

Recent lattice calculations of a_0^2

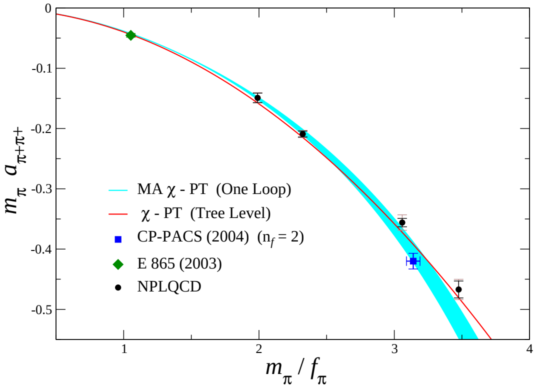
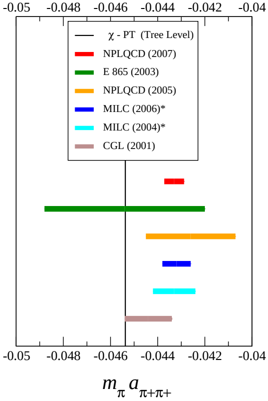


Figure from NPLQCD, arXiv:0706.3026

cf. Talk by A. Walker-Loud

Recent lattice calculations of a_0^2



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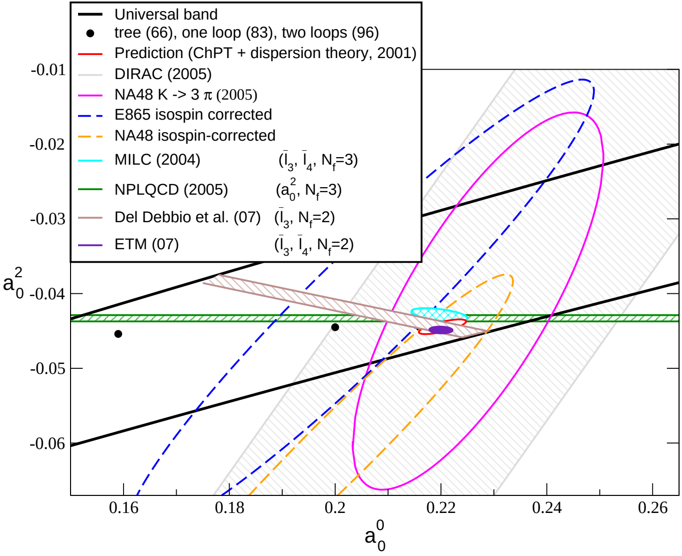
Recent lattice calculations of a_0^2



Conclusions

- Recent progress in approaching the chiral region on the lattice is impressive
- Statistical errors on the low-energy constants determined on the lattice are tiny – we see already the enormous potential of the method
- Study of systematic effects (continuum extrapolation, finite volume, higher chiral orders) is only at the beginning
  Need to exclude that "the customer is always right"
  P.Boyle, Lat07
- (Near) future:
  1. make an effort community-wide to determine as accurately as possible the low-energy constants
  2. use this input in CHPT to describe the phenomenology $\Rightarrow$ test of QCD
- $\Rightarrow$ ongoing discussion within the Flavianet Lattice Working Group – if you have ideas or proposals, they are most welcome: contact me or Rainer Sommer