QCD at low energy: the simplicity of complex non-perturbative phenomena

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Lecture II: Higher orders, applications

Why go beyond $O(p^2)$? Why loops?

$$\langle 0|Te^{\mathrm{i}\int d^4x\mathcal{L}}|0\rangle = e^{\mathrm{i}Z[v,a,s,p]} = \mathcal{N}^{-1}\int [dU] e^{\mathrm{i}\int d^4x\mathcal{L}_{\mathrm{eff}}}$$

Why go beyond $O(p^2)$? Why loops?

- Why not? Chiral Symmetry forbids $O(p^0)$ interactions between pions, but allows all higher orders
- Unitarity requires that if an amplitude at order p^2 is purely real, at order p^4 its imaginary part is nonzero.
  Take the $\pi\pi$ scattering amplitude. The elastic unitarity relation for the partial waves t_ℓ^I of isospin I and angular momentum ℓ reads:

$$Im\, t_\ell^I = \sqrt{1 - \frac{4M_\pi^2}{s}} |t_\ell^I|^2$$

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- *The correct imaginary parts are generated automatically by loops*
- *The divergences occuring in the loops can be disposed of just like in a renormalizable field theory*

Effective quantum field theory

The method of effective quantum field theory provides a rigorous framework to compute Green functions that respect all the good properties we require: symmetry, analyticity, unitarity

The method yields a systematic expansion of the Green functions in powers of momenta and quark masses

In the following I will discuss in detail how this works when you consider loops:

- I will consider the finite, analytically nontrivial part of the loops and discuss in detail its physical meaning
- I will consider the divergent part of the loops and discuss how the renormalization program works

Scalar form factor of the pion

$$\langle \pi^i(p_1)\pi^j(p_2)|\hat{m}(\bar{u}u + \bar{d}d)|0\rangle =: \delta^{ij}\Gamma(t) \ , \quad t = (p_1 + p_2)^2 \ ,$$

At tree level:

$$\Gamma(t) = 2\hat{m}B = M_\pi^2 + O(p^4) \ ,$$

in agreement with the Feynman–Hellman theorem:
the expectation value of the perturbation in an eigenstate of the total Hamiltonian determines the derivative of the energy level with respect to the strength of the perturbation:

$$\hat{m}\frac{\partial M_\pi^2}{\partial \hat{m}} = \langle \pi|\hat{m}\bar{q}q|\pi\rangle = \Gamma(0) \ .$$

This matrix element is relevant for the decay $h \to \pi\pi$, which, for a light Higgs would have been the main decay mode

Donoghue, Gasser & Leutwyler (90)

Dispersion relation for $\Gamma(t)$

For $t \geq 4M_\pi^2$ $\mathrm{Im}\,\Gamma(t) \neq 0$. $\Gamma(t)$ is analytic everywhere else in the complex t plane, and obeys the following dispersion relation: $\bar{\Gamma}(t) = \Gamma(t)/\Gamma(0)$

$$\bar{\Gamma}(t) = 1 + bt + \frac{t^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{dt'}{t'^2} \frac{\mathrm{Im}\,\bar{\Gamma}(t')}{t' - t}$$

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Unitarity implies $[\sigma(t) = \sqrt{1 - 4M_\pi^2/t}]$

$$\mathrm{Im}\,\bar{\Gamma}(t) = \sigma(t)\bar{\Gamma}(t)t_0^{0*}(t) = \bar{\Gamma}(t)e^{-i\delta_0^0}\sin\delta_0^0 = |\bar{\Gamma}(t)|\sin\delta_0^0$$

where t_0^0 is the S–wave, $I = 0$ $\pi\pi$ scattering amplitude

Strictly speaking, the above unitarity relation is valid only for $t \leq 16M_\pi^2$. To a good approximation, however, it holds up to the $K\bar{K}$ threshold

Dispersion relation and chiral counting

$$
\begin{aligned}
\bar{\Gamma}(t) &= 1 + bt + \frac{t^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{dt'}{t'^2} \frac{|\bar{\Gamma}(t')| \sin\delta_0^0(t')}{t'-t} \\
b &\sim O(1)\left(1 + O(M_\pi^2)\right) \\
\delta_0^0 &\sim O(p^2)\left(1 + O(p^2)\right)
\end{aligned}
$$

There are two $O(p^2)$ correction to $\bar{\Gamma}$:

1. $O(1)$ contribution to b;
2. the dispersive integral containing the $O(p^2)$ phase δ_0^0.

Notice that the latter is fixed by unitarity and analyticity

Are these respected by the one loop calculation?

Dispersion relation and one–loop CHPT

The full one–loop expression of $\bar{\Gamma}(t)$ reads as follows:

$$\bar{\Gamma}(t) = 1 + \frac{t}{16\pi^2 F_\pi^2}(\bar{l}_4 - 1) + \frac{2t - M_\pi^2}{2F_\pi^2}\bar{J}(t)$$

$$\bar{J}(t) = \frac{1}{16\pi^2}\left[\sigma(t)\ln\frac{\sigma(t)-1}{\sigma(t)+1} + 2\right]$$

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To prove that unitarity and analyticity are respected at this order is sufficient to add:

$$\delta_0^0(t) = \sigma(t)\frac{2t - M_\pi^2}{32\pi F_\pi^2} + O(p^4)$$

$$\bar{J}(t) = \frac{t}{16\pi^2}\int_{4M_\pi^2}^{\infty}\frac{dt'}{t'}\frac{\sigma(t')}{t'-t}$$

Can you prove it?

Hints:

- Subtract $\bar{J}(t)$ once more

$$\bar{J}(t) = \frac{t}{96\pi^2 M_\pi^2} + \frac{t^2}{16\pi^2} \int_{4M_\pi^2}^{\infty} \frac{dt'}{t'^2} \frac{\sigma(t')}{t' - t}$$

- Trick to pull out a linear term from the dispersive integral:

$$\int_{4M_\pi^2}^{\infty} \frac{dt'}{t'^2} \frac{t'\sigma(t')}{t' - t} = t \int_{4M_\pi^2}^{\infty} \frac{dt'}{t'^2} \frac{\sigma(t')}{t' - t} + \int_{4M_\pi^2}^{\infty} \frac{dt'}{t'^2} \sigma(t')$$

High-energy contributions

The dispersive integral goes up to $s' = \infty$, but the integrand is correct only at low energy!

$$
\begin{aligned}
\bar{\Gamma}(t)_{h.e.} &= \frac{t^2}{\pi}\int_{\Lambda^2}^{\infty}\frac{dt'}{t'^2}\frac{|\bar{\Gamma}(t')|\sin\delta_0^0(t')}{t'-t} \\
&\sim \frac{t^2}{\pi}\int_{\Lambda^2}^{\infty}\frac{dt'}{t'^2}|\bar{\Gamma}(t')|\sin\delta_0^0(t')\frac{1}{t'}\left(1+\frac{t}{t'}+\ldots\right) \\
&\sim ct^2+\mathcal{O}(t^3)
\end{aligned}
$$

The contributions from the high-energy region of the dispersive integral are formally of higher order – introducing a cut-off to remove them would only make the formulae more cumbersome

Renormalization at one loop

$$\int \frac{d^4l}{(2\pi)^4} \frac{\{p^2, p\cdot l, l^2\}}{(l^2 - M^2)((p-l)^2 - M^2)} \,, \qquad p = p_1 + p_2$$

$$\sim \underbrace{\int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2 - M^2)}}_{T(M^2)} + p^2 \underbrace{\int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2 - M^2)((p-l)^2 - M^2)}}_{J(p^2)}$$

$$T(M^2) = a + bM^2 + \bar{T}(M^2) \qquad J(t) = J(0) + \bar{J}(t)$$

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$$T(M^2) = a + bM^2 + \bar{T}(M^2) \qquad J(t) = J(0) + \bar{J}(t)$$

$\bar{T}(M^2)$ and $\bar{J}(t)$ are finite

$$\Gamma(t) \sim M^2 \left[1 + \underbrace{bM^2 + tJ(0)}_{\text{divergent part}} + \bar{T}(M^2) + \bar{J}(t) \right]$$

Counterterms

$$\mathcal{L}_2 \quad \Rightarrow \quad \Gamma^{(2)}(t) \sim M^2$$

$$\mathcal{L}_4 \quad \Rightarrow \quad \Gamma^{(4)}(t) \sim \ell_3 M^4 + \ell_4 M^2 t$$

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To remove the divergences one only needs to properly define the couplings ($\ell_{3,4}$) in the lagrangian at order $O(p^4)$

Quote from Weinberg's book on QFT, vol. I: "(...) as long as we include every one of the infinite number of interactions allowed by symmetries, the so–called non–renormalizable theories are actually just as renormalizable as renormalizable theories."

Chiral logarithms

Scalar radius of the pion

$$\Gamma(t) = \Gamma(0)\left[1 + \frac{1}{6}\langle r^2\rangle_S^\pi t + O(t^2)\right]$$

$$\langle r^2\rangle_S^\pi \sim J(0) = \int \frac{d^4l}{(2\pi)^4}\frac{1}{(l^2 - M^2)^2} \sim \ln\frac{M^2}{\Lambda^2}$$

The integral is UV divergent, but also IR divergent if $M \to 0$:

$$\lim_{M^2\to 0}\langle r^2\rangle_S^\pi \sim \ln M^2 \quad ,$$

The extension of the cloud of pions surrounding a pion (or any other hadron) goes to infinity if pions become massless (Li and Pagels '72)

Chiral symmetry and renormalization

To remove the divergent part in $\Gamma(t)$ we have to fix the divergent part of chiral–invariant operator of order $O(p^4)$

e.g. $\langle \partial_\mu U^\dagger \partial^\mu U \rangle \langle B\mathcal{M}(U + U^\dagger) \rangle \sim \ldots + M^2 \phi^2 \partial_\mu \phi^4 \partial^\mu \phi^6 + \ldots$

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Chiral symmetry implies that after calculating the divergent part of $\Gamma(s)$ I also know the divergent part of the $6\pi \to 6\pi$ scattering amplitude

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1. Do we have a proof that quantum effects do not introduce violations of the chiral symmetry? Or that one can build a chiral invariant generating functional only with a path integral over a chiral invariant classical action?

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1. Do we have a proof that quantum effects do not introduce violations of the chiral symmetry? Or that one can build a chiral invariant generating functional only with a path integral over a chiral invariant classical action?
2. Is there a tool that allows one to calculate the divergences keeping chiral invariance explicit in every step of the calculation?

Leutwyler's theorem

What is the most general way of constructing a chiral-invariant generating functional out of a path integral over the Goldstone boson degrees of freedom?

$$Z[v',a',s',p'] = Z[v,a,s,p] \Leftrightarrow \mathcal{L}_{\rm eff}[v',a',s',p'] = \mathcal{L}_{\rm eff}[v,a,s,p]?$$

For Lorentz–invariant theories in 4 dimensions, a path integral constructed with gauge–invariant lagrangians is a necessary and sufficient condition to obtain a gauge–invariant generating functional

The theorem also includes the case in which the symmetry is anomalous and the case in which the symmetry is explicitly broken

Leutwyler (94), d'Hoker & Weinberg (94)

Chiral invariant renormalization

- Gasser & Leutwyler (84) have shown that, using the background field method and heat kernel techniques, the calculation of the divergences at one loop – and the corresponding renormalization – can be performed in an explicitly chiral invariant manner
- The method has been extended and applied to two loops (Bijnens, GC & Ecker 98). After a long and tedious calculation, the divergent parts of all the counterterms at $\mathcal{O}(p^6)$ has been provided
- The renormalization of CHPT up to two loops has been performed explicitly: the calculation of any amplitude at two loops can be immediately checked by comparing the divergent part of Feynman diagrams to the divergent parts of the relevant counterterms

Chiral perturbation theory

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  symmetry, analyticity, unitarity
- The method yields a systematic expansion of the Green functions in powers of momenta and quark masses
- The method has been rigorously established and can be formulated as a set of calculational rules:

LO	tree level diagrams with $\mathcal{L}_2$
NLO	tree level diagrams with $\mathcal{L}_4$
	1-loop diagrams with $\mathcal{L}_2$
NNLO	tree level diagrams with $\mathcal{L}_6$
	2-loop diagrams with $\mathcal{L}_2$
	1-loop diagrams with one vertex from $\mathcal{L}_4$

$\pi\pi$ scattering at NLO

$$
\begin{aligned}
a_0^0 &= \frac{7M_\pi^2}{32\pi F_\pi^2}\left[1 + \frac{M_\pi^2}{3}\langle r^2\rangle_S^\pi + \frac{200\pi F_\pi^2 M_\pi^2}{7}(a_2^0 + 2a_2^2)\right. \\
&- \left.\frac{M_\pi^2}{672\pi^2 F_\pi^2}(15\bar{\ell}_3 - 353)\right] = 0.16 \cdot 1.25 = 0.20 \\
2a_0^0 - 5a_0^2 &= \frac{3M_\pi^2}{4\pi F_\pi^2}\left[1 + \frac{M_\pi^2}{3}\langle r^2\rangle_S^\pi + \frac{41M_\pi^2}{192\pi^2 F_\pi^2}\right] = 0.624
\end{aligned}
$$

Gasser and Leutwyler (83)

Higher orders too large?

Higher order corrections are suppressed by $\mathcal{O}(p^2/\Lambda^2)$
$\Lambda \sim 1$ GeV $\Rightarrow$ expected to be a few percent

$$a_0^0 = 0.200 + \mathcal{O}(p^6) \qquad a_0^2 = -0.0445 + \mathcal{O}(p^6)$$

The reason for the rather large correction in a_0^0 is a chiral log

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2}\left[1 + \frac{9}{2}\ell_\chi + \ldots\right] \qquad a_0^2 = -\frac{M_\pi^2}{16\pi F_\pi^2}\left[1 - \frac{3}{2}\ell_\chi + \ldots\right]$$

$$\ell_\chi = \frac{M_\pi^2}{16\pi^2 F_\pi^2} \ln \frac{\mu^2}{M_\pi^2}$$

Gasser and Leutwyler (84)

Scattering lengths: theory vs experiment

- Weinberg (66), $O(p^2)$:

$$a_0^0 = 0.16, \qquad a_0^2 = -0.045$$

- Gasser and Leutwyler (83), $O(p^4)$:

$$a_0^0 = 0.20 \pm 0.01, \qquad a_0^2 = -0.044$$

- Bijnens, GC, Ecker, Gasser and Sainio (95), $O(p^6)$:

$$a_0^0 = 0.217, \qquad a_0^2 = -0.044$$

- GC, Gasser and Leutwyler (01), $O(p^6)$+dispersion relations:

$$a_0^0 = 0.220 \pm 0.005, \qquad a_0^2 = -0.0444 \pm 0.0010$$

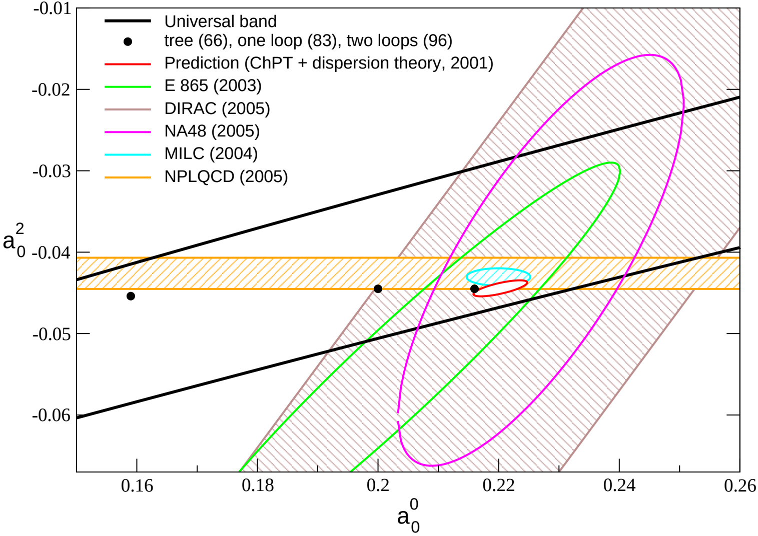
Scattering lengths: theory vs experiment

$a_0^0 = 0.26 \pm 0.05$		Rosselet et al. (77)
$a_0^0 = 0.216 \pm 0.013 \pm 0.003$		Pislak et al. (E865) (03)
$\lvert a_0^0 - a_0^2 \rvert = 0.264 \,^{+0.033}_{-0.020}$		Adeva et al. (DIRAC) (05)
$a_0^0 - a_0^2 = 0.268 \pm 0.010 \pm 0.013$		Batley et al. (NA48/2) (06)
$a_0^0 = 0.256 \pm 0.011$	[PRELIMINARY]	B. Bloch-Devaux (NA48/2) (06)

Method of measurement

Rosselet et al.	K_{e4}	Cabibbo & Maksymowicz (65)
Pislak et al. (E865)	" "	" "
Adeva et al. (DIRAC)	Pionium	Deser et al. (56)
Batley et al. (NA48/2)	$K \to 3\pi$	Cabibbo (04)

Scattering lengths: theory vs experiment



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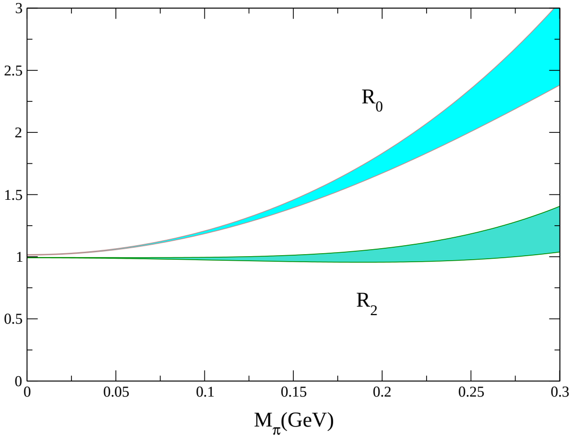


M_π-dependence of the scattering lengths: $R_I = a_0^I / a_0^I(\text{Weinberg})$

Sensitivity to the quark condensate

The constant $\bar{\ell}_3$ appears in the chiral expansion of the pion mass

$$M_\pi^2 = 2B\hat{m}\left[1 + \frac{2B\hat{m}}{16\pi F_\pi^2}\bar{\ell}_3 + \mathcal{O}(\hat{m}^2)\right]$$

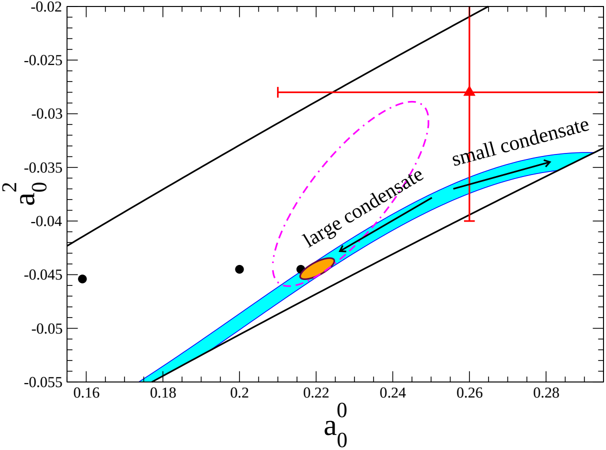
$$\hat{m} = \frac{m_u + m_d}{2} \qquad B = -\frac{1}{F^2}\langle 0|\bar{q}q|0\rangle$$

Its size tells us what fraction of the pion mass is given by the Gell-Mann–Oakes–Renner term

$$M_{\rm GMOR}^2 \equiv 2B\hat{m}$$

Crude estimate: $\bar{\ell}_3 = 2.9 \pm 2.4$ Gasser & Leutwyler (84)

Sensitivity to the quark condensate



The E865 data on $K_{\ell 4}$ imply that

GC, Gasser & Leutwyler PRL (01)

$$M_{\mathrm{GMOR}} > 94\% M_\pi$$

$\pi\pi$ scattering, Roy equations

- Crossing symmetry implies that $\mathrm{Re}T(s,t)$ is given by a twice subtracted dispersive integral over $\mathrm{Im}T(s,t)$ in the physical region S.M. Roy 1971
- As subtraction constants one can choose the S-wave scattering lengths

$$a_0^0\,,\quad a_0^2$$

- Projecting onto the partial waves one obtains the

Roy equations

coupled integral equations for the partial waves
- Pioneering work in solving numerically these equations has been performed in the seventies Basdevant, Froggatt, Petersen 1974

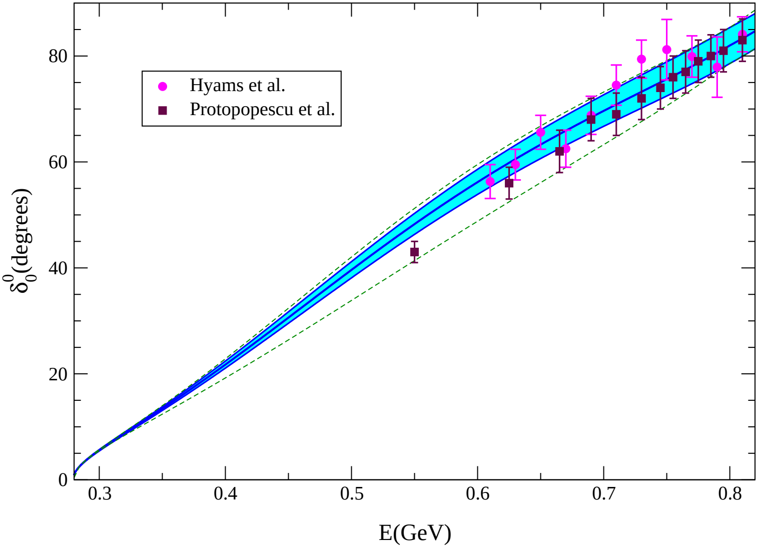
Roy equations and chiral symmetry

- Two subtractions $\Rightarrow$ dispersive integrals converge rapidly at low energy the most important ingredient are the scattering lengths [not well known in the seventies]
- Phenomenological information on the imaginary parts above 0.8 GeV, though not precise, has little impact on the uncertainties at low energy Ananthanarayan, GC, Gasser, Leutwyler (01)

  Descotes, Fuchs, Girlanda, Stern (02)
- Chiral symmetry provides precise information about the scattering lengths

  Weinberg (66), Gasser & Leutwyler (84), Bijnens, GC, Ecker, Gasser, Sainio (96)
- Matching the dispersive and chiral representation near $s = 0$ one obtains the $\pi\pi$ scattering amplitude at low energy to a high degree of precision GC, Gasser and Leutwyler (01)

Roy equations and chiral symmetry



The σ in the PDG

: S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004) and 2005 partial update for edition 2006 (URL: http://pdg.lbl.gov)

$f_0(600)$ or σ

$I^G(J^{PC}) = 0^+(0^{++})$

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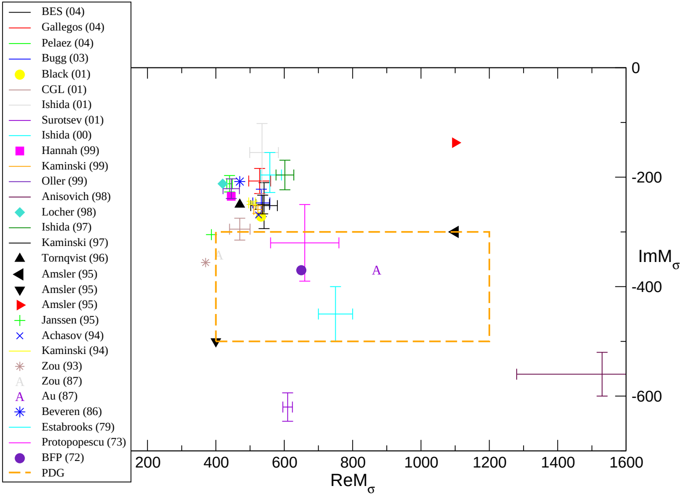
$f_0(600)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma \approx 2\ \mathrm{Im}(\sqrt{s_{\mathrm{pole}}})$.

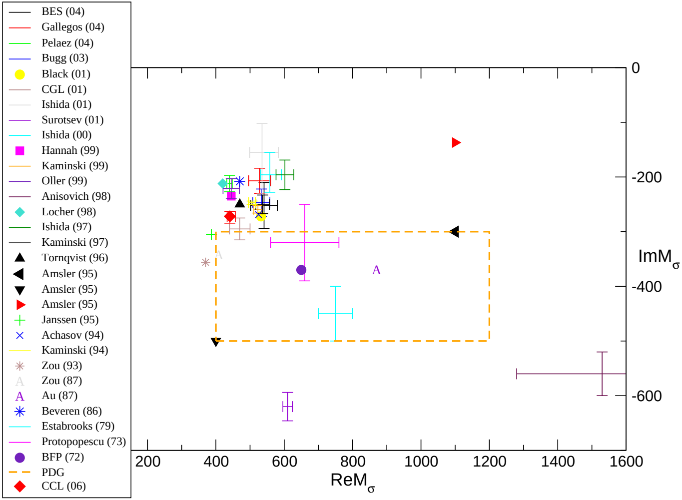
VALUE (MeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
(400–1200)−*i*(300–500) OUR ESTIMATE				
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$(541 \pm 39)-i(252 \pm 42)$	[1] ABLIKIM	04A	BES2	$J/\psi \to \omega\pi^+\pi^-$
$(528 \pm 32)-i(207 \pm 23)$	[2] GALLEGOS	04	RVUE	Compilation
$(440 \pm 8)-i(212 \pm 15)$	[3] PELAEZ	04A	RVUE	$\pi\pi \to \pi\pi$
$(533 \pm 25)-i(247 \pm 25)$	[4] BUGG	03	RVUE	
$532 - i272$	BLACK	01	RVUE	$\pi^0\pi^0 \to \pi^0\pi^0$
$(470 \pm 30)-i(295 \pm 20)$	[5] COLANGELO	01	RVUE	$\pi\pi \to \pi\pi$
$(535^{+48}_{-36})-i(155^{+76}_{-53})$	[6] ISHIDA	01		$\Upsilon(3S) \to \Upsilon\pi\pi$
$610 \pm 14 - i620 \pm 26$	[7] SUROVTSEV	01	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(558^{+34}_{-27})-i(196^{+32}_{-41})$	ISHIDA	00B		$p\overline{p} \to \pi^0\pi^0\pi^0$
$445 - i235$	HANNAH	99	RVUE	π scalar form factor
$(523 \pm 12)-i(259 \pm 7)$	KAMINSKI	99	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\sigma\sigma$
$442 - i\ 227$	OLLER	99	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$469 - i203$	OLLER	99B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$445 - i221$	OLLER	99C	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$
$(1530^{+\ 90}_{-250})-i(560 \pm 40)$	ANISOVICH	98B	RVUE	Compilation
$420 - i\ 212$	LOCHER	98	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(602 \pm 26)-i(196 \pm 27)$	[8] ISHIDA	97		$\pi\pi \to \pi\pi$
$(537 \pm 20)-i(250 \pm 17)$	[9] KAMINSKI	97B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, 4π
$470 - i250$	[10,11] TORNQVIST	96	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $K\pi$, $\eta\pi$
$\sim (1100 - i300)$	AMSLER	95B	CBAR	$\overline{p}p \to 3\pi^0$
$400 - i500$	[11,12] AMSLER	95D	CBAR	$\overline{p}p \to 3\pi^0$
$1100 - i137$	[11,13] AMSLER	95D	CBAR	$\overline{p}p \to 3\pi^0$
$387 - i305$	[11,14] JANSSEN	95	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$525 - i269$	[15] ACHASOV	94	RVUE	$\pi\pi \to \pi\pi$
$(506 \pm 10)-i(247 \pm 3)$	KAMINSKI	94	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$370 - i356$	[16] ZOU	94B	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$408 - i342$	[11,16] ZOU	93	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$870 - i370$	[11,17] AU	87	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$470 - i208$	[18] BEVEREN	86	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$, ...
$(750 \pm 50)-i(450 \pm 50)$	[19] ESTABROOKS	79	RVUE	$\pi\pi \to \pi\pi$, $K\overline{K}$
$(660 \pm 100)-i(320 \pm 70)$	PROTOPOP...	73	HBC	$\pi\pi \to \pi\pi$, $K\overline{K}$
$650 - i370$	[20] BASDEVANT	72	RVUE	$\pi\pi \to \pi\pi$

The σ in the PDG



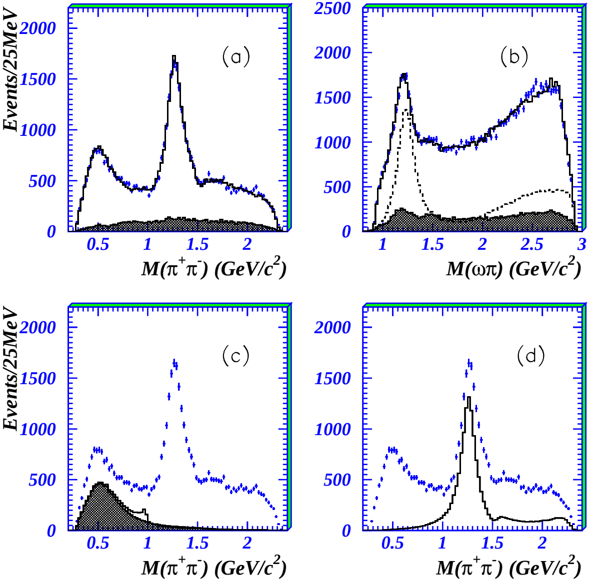
“Is there any reason why composite $\bar{q}q$ or $\bar{\ell}\ell$ scalar particles have never been clearly established?”

The σ in the PDG



CCL = Caprini, GC, Leutwyler PRL (06)

The σ in the data – BES (04), $J/\psi \to \omega\pi^+\pi^-$



How is the σ pole determined?

The relevant question is:

Where does the amplitude have a pole on the second Riemann sheet of the complex s plane?

The answer ought to be model- and parametrization-independent

How is the σ pole determined?

What is usually done is instead the following:

Fit the data with a parametrization, e.g.

$$
\begin{aligned}
f &= \frac{G_\sigma}{M^2 - s - iM\Gamma_{\rm tot}(s)} \\
\Gamma_{\rm tot}(s) &= g_1 \frac{\rho_{\pi\pi}(s)}{\rho_{\pi\pi}(M^2)} + g_2 \frac{\rho_{4\pi}(s)}{\rho_{4\pi}(M^2)}
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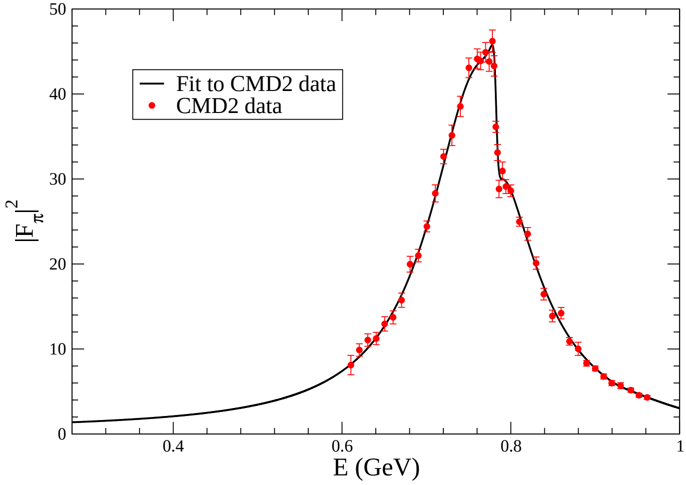
The fit to the data determines the σ parameters, M and $\Gamma_{\rm tot}$

The outcome is parametrization-dependent
Moreover, an obvious shortcoming of many of the parametrizations used to fit data is the neglect of the left-hand cut

Compare to the ρ in $e^+e^- \to \pi^+\pi^-$



Roy representation of t_0^0

Double-subtracted, crossing symmetric dispersion relation for t_0^0

$$t_0^0(s) = a + (s - 4M_\pi^2)\, b + \int_{4M_\pi^2}^{\Lambda^2} ds' \left\{ K_0(s,s')\, \mathrm{Im}\, t_0^0(s') + K_1(s,s')\, \mathrm{Im}\, t_1^1(s') + K_2(s,s')\, \mathrm{Im}\, t_0^2(s') \right\} + d_0^0(s)$$

$$a = a_0^0\,, \quad b = (2\, a_0^0 - 5\, a_0^2)/(12 M_\pi^2)$$

$$K_0(s,s') = \frac{1}{\pi(s'-s)} + \frac{2 \ln((s+s'-4M_\pi^2)/s')}{3\pi(s-4M_\pi^2)} - \frac{5s'+2s-16M_\pi^2}{3\pi s'(s'-4M_\pi^2)}$$

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This representation allows one to evaluate t_0^0 in the complex plane – in its domain of validity on the first sheet.

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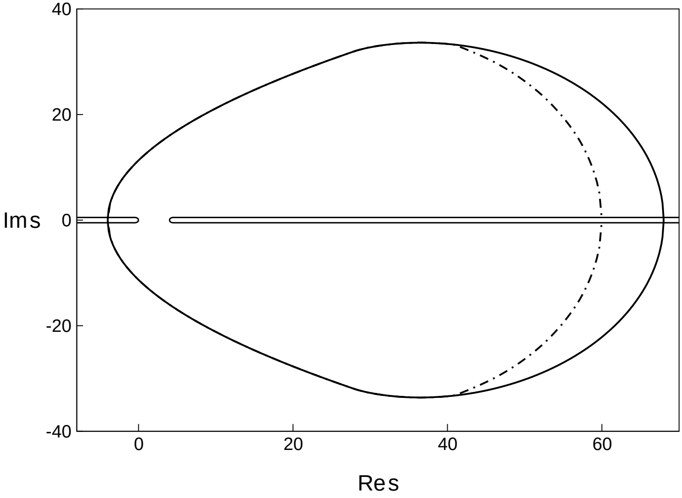

Caprini, GC, Leutwyler (05)

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This representation allows one to evaluate t_0^0 in the complex plane – in its domain of validity on the first sheet.

Poles, however, are to be found on the second sheet

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The second sheet is reached by analytic continuation crossing the real axis from above: (for ϵ infinitesimally small)

$$S_0^{0\,II}(s-i\epsilon) = S_0^{0\,I}(s+i\epsilon) = \left[S_0^{0\,I}(s-i\epsilon)\right]^{-1}$$

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By analytic continuation, it is then true everywhere that

$$S_0^{0\,II}(s) = \left[S_0^{0\,I}(s)\right]^{-1}$$

Poles on the second sheet correspond to zeros on the first sheet!

Caprini, GC and Leutwyler, PRL (06)

Summary: method to determine the pole position

- Roy equations provide an explicit representation of t_0^0 on the first sheet, in terms of the imaginary parts of the partial waves on the real axis and two subtraction constants:

$$t_0^0(s) = a + (s - 4M_\pi^2)\, b + \int_{4M_\pi^2}^{\Lambda^2} ds' K_0(s, s')\, \mathrm{Im}\, t_0^0(s') + \ldots$$

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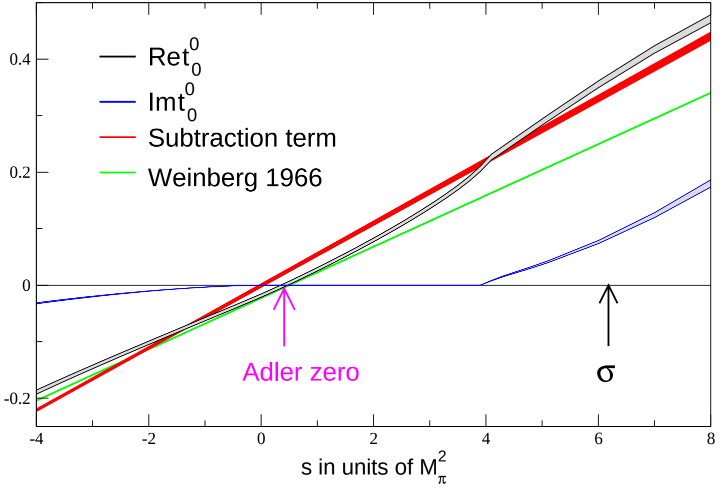
- Using as input the imaginary parts of the partial waves and the two S-wave scattering lengths one can determine the position of the poles of the S-matrix on the second sheet

Importance of the scattering lengths

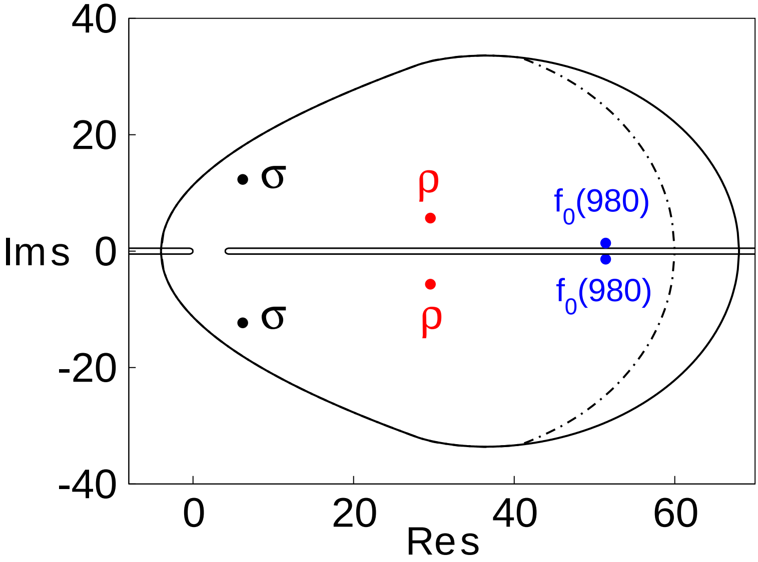


Zeros of S_0^0 (and S_1^1)

Input: the imaginary parts from Roy solutions below 1.15 GeV [GC, Leutwyler, in preparation] and the central values of the two scattering lengths (CHPT) we find two pairs of zeros

$$m_\sigma^2 = (6.2 \pm i\,12.3)\,M_\pi^2 \qquad m_{f_0}^2 = (51.4 \pm i\,1.4)\,M_\pi^2$$

Zeros of S_0^0 (and S_1^1)



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Error analysis: [at fixed a_0^0, a_0^2 and $\delta_A \equiv \delta_0^0(0.8\text{GeV})$]

$$m_\sigma = 441 \pm 4 - i(272 \pm 6)\ \text{MeV}$$

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$$m_\sigma = 441 \pm 7 - i272 \pm 9$$

Caprini, GC and Leutwyler, PRL (06)

Different inputs

- The extension of the Roy equation analysis from 0.8 to 1.15 GeV has no impact on m_σ. Using CGL (01) we get

$$
\begin{aligned}
m_\sigma^{\mathrm{CGL}}(\text{model indep.}) &= 439.4 - i274.5 \text{ MeV} \\
m_\sigma^{\mathrm{CGL}}(\text{param.-dep.}) &= 470 \pm 30 - i295 \pm 20 \text{ MeV}
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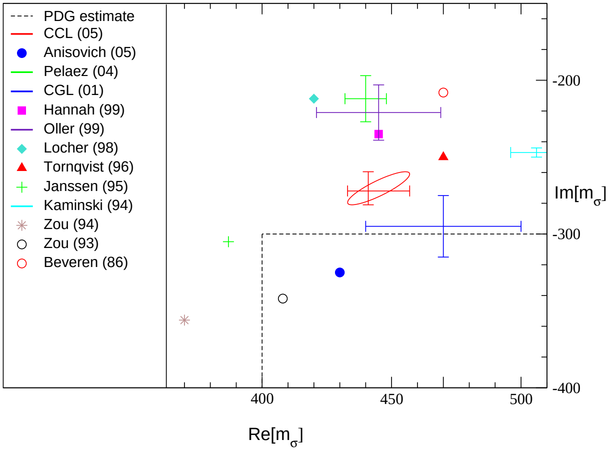
- Using a phenomenological representation of the $\pi\pi$ scattering amplitude [Peláez and Ynduráin (05)] we obtain

$$m_\sigma^{\rm PY} = 445 - i241 \text{ MeV}$$

Our formula which describes the dependence on the main three input parameters reproduces this result:

$$\begin{aligned} a_0^0(PY) &= 0.23\,, \quad a_0^2(PY) = -0.048\,, \quad \delta_A(PY) = 90.9^\circ \\ &\Rightarrow m_\sigma = 447 - i242 \text{ MeV} \end{aligned}$$

Comparison to PDG and experimental information



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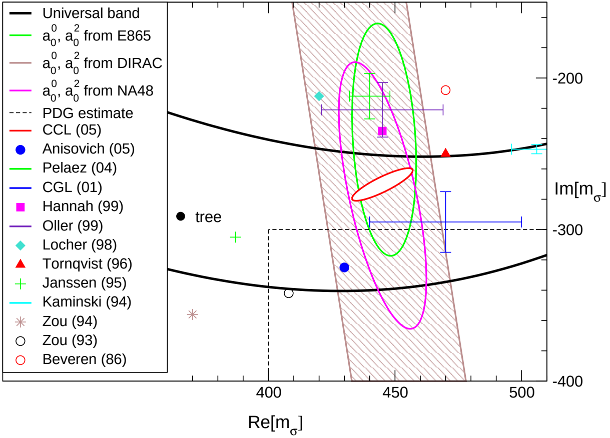

Areas of application

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  - Purely strong interactions ((semi)leptonic decays)

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  - Two nucleon sector: NN scattering, nuclear forces

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  - Extrapolation to zero lattice spacing

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- Condensed–matter systems with spontaneous symmetry breaking

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- Condensed–matter systems with spontaneous symmetry breaking
  - Ferromagnets
  - Antiferromagnets in $d = 3$ or $d = 2$

Areas of application

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- Condensed–matter systems with spontaneous symmetry breaking
- Electroweak symmetry breaking – models in which the electroweak symmetry is broken strongly
- General relativity as an effective field theory
- Falsification of string theory?!?!

Areas of application

UCSD/PTH-06-06, UTTTG-06-06, CMUHEP-06-0

Falsifying String Theory Through WW Scattering

Jacques Distler,[1,*] Benjamin Grinstein,[2,†] and Ira Z. Rothstein[3,‡]
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[2] *University of California, San Diego, Dept. of Physics, La Jolla, California 92093-0319, USA*
[3] *Carnegie-Mellon University, Dept. of Physics, Pittsburgh, Pennsylvania 15213, USA*

We show that the coefficients of operators in the electroweak chiral Lagrangian can be bounded if the underlying theory obeys the usual assumptions of Lorentz invariance, analyticity and unitarity for all scales. Violations of these bounds can be explained by either the existence of new physics below the naive cut-off of the the effective theory, or by the breakdown of one of these assumptions in the short distance theory. If no light resonances are found, then a measured violation of the bound would falsify string theory.

Summary

- The finite, analytically nontrivial part of the one loop integrals automatically generates the correct imaginary parts, as required by unitarity.
- Effective quantum field theory is a systematic method to generate a perturbative solution of dispersion relations
- The UV divergences encountered in loop integrals can be removed according to standard renormalization methods
- Some loop integrals have also an IR singular behaviour which has a very clear physical meaning, and again shows the necessity of taking loop effects into account
- Leutwyler's theorem: doing a path integral over an effective Lagrangian is the most general way to construct an invariant generating functional
- I have illustrated the method discussing two applications:
  - the $\pi\pi$ S-wave scattering lengths
  - the determination of the σ pole position